Comment on "Fundamentals of pair diffusion in kinematic simulations of turbulence"

B. J. Devenish and D. J. Thomson
Met Office, FitzRoy Road, Exeter EX1 3PB, United Kingdom

(Received 8 January 2008; revised manuscript received 28 July 2009; published 23 October 2009)

Osborne *et al.* [Phys. Rev. E **74**, 036309 (2006)] suggested that our conclusions on the ability of kinematic simulation to represent the t^3 Richardson law [Thomson and Devenish, J. Fluid Mech. **526**, 277 (2005)] were an artifact of our choice of time step. Here we repeat some of the simulations with a small fixed time step, enabling us to confirm that our previous study was not compromised by the variable time step used.

DOI: 10.1103/PhysRevE.80.048301 PACS number(s): 47.27.E−, 47.27.Gs

Recently we argued that, in kinematic simulation of three-dimensional turbulence with an inertial subrange spectrum, the mean-square pair separation does not follow the Richardson law $\langle r^2\rangle \sim \varepsilon t^3$ expected in real flows (see Ref. [1]). (Here ε is the notional energy dissipation rate per unit mass in the kinematic simulation, r is the particle separation, and t is time.) Instead we argued that, because of the lack of "sweeping" (i.e., advection) of the small scale modes by larger modes in kinematic simulation, the separation should, in the limit of a long inertial subrange, grow like t^6 if a strong mean flow is present and like $t^{9/2}$ for no mean flow. These predictions were supported by numerical simulations. Note that, by a strong mean flow, we mean that a strong mean flow is simply added to the velocity field without advection of the eddies by the mean flow. This is clearly unrealistic but exaggerates the sweeping problem and is useful to help understand the limitations of kinematic simulation as a realistic model for turbulence.

The simulations which were carried out in support of our arguments used an adaptive time step in calculating the particle pair trajectories. This time step was determined separately for each pair and depended on the pair separation. A justification was offered for the choice of time step: the eddies much smaller than the pair separation have negligible effect on the separation process and so do not need to be resolved by the time step. We believe this is correct and it was supported by some simulations using a fixed time step small enough to resolve the smallest eddies. However, these simulations, for reasons of computational cost, extended over only a small fraction of the time required for the pair separation to reach the integral scale.

Osborne *et al.* Ref. [2] present simulations with adaptive and fixed small time steps which show significant differences and conclude that our results may be compromised by our choice of time step. See also the discussion in Refs. [3,4]. In this Comment we repeat (i) some of the simulations of Ref. [1] with the adaptive time step replaced by a fixed small time step and (ii) the relevant simulations of Ref. [2], in order to test Osborne *et al.*'s conclusion.

The method used to generate the random flows is identical to that used in Ref. [1]. The resulting flows are superpositions of independent random Fourier modes with an energy spectrum proportional to $\varepsilon^{2/3}k^{-5/3}$ for $2\pi/L \leq k \leq 2\pi/\eta$, where k is the wave number and L and η are proportional to the integral length scale and the Kolmogorov dissipation length scale, respectively. In some simulations a mean velocity $(\bar{U},0,0)$ is added to the flow. In the following we use σ_u to denote the rms value of any one component of the velocity fluctuations and λ to denote the time-dependency parameter (defined in Ref. [1]).

Particle pairs are released with separation r_0 and tracked through the flow using a forward Euler method. In the adaptive time-step simulations presented in Ref. [1], the time step was given by

$$\Delta t = \min\left(0.1\frac{\min(r,L)}{\max(\bar{U},\sigma_u)},\quad 0.01\frac{[\min(r,L)]^{2/3}}{\lambda\sigma_u/L^{1/3}}\right). \tag{1}$$

Reference [1] argued that this time step is small enough to resolve the changes in particle velocity due to (i) the sweeping of particles through the eddies that dominate the separation process and (ii) the temporal change of such eddies caused by λ [note that the time scale for sweeping a pair with separation r through an eddy of size $\sim r$, i.e., $r/\max(\bar{U},\sigma_u)$, is, for $r \ll L$, much smaller than the expected time scale of the physics in real Navier-Stokes flows, which is of order $r^{2/3}/\varepsilon^{1/3}$]. Also Ref. [1] presented some sensitivity tests and comparisons with short-duration fixed-time-step simulations in support of the choice of time step. The longer-duration fixed-time-step simulations presented here (we call these the "new" simulations below) are designed to test this more rigorously. For these simulations, we use a time step given by

$$\Delta t = \min\left(0.1\frac{\eta}{\max(\bar{U},\sigma_u)},\quad 0.01\frac{\eta^{2/3}}{\lambda\sigma_u/L^{1/3}}\right). \tag{2}$$

This should be small enough to resolve the effect of even the smallest eddies throughout the evolution of the pair separation.

For the adaptive time-step simulations presented in Ref. [1], a number of pairs were tracked in each realization of the flow. For the new fixed time-step versions of these simulations we only follow one pair in each realization of the velocity field to maximize the statistical accuracy for the available computing time (following a pair is far more expensive than generating a flow field, and so it makes sense to ensure the pairs are completely independent).

Figures 1 and 2 show comparisons between the adaptive time-step simulations presented in Ref. [1] and the new fixed time-step simulations for two frozen flow cases. Although the new simulations cover a shorter time than the adaptive time-step simulations, they extend a decade further than the fixed time-step simulations of Ref. [1] and are long enough

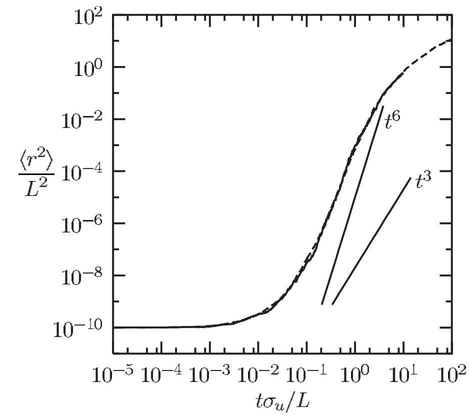

FIG. 1. Comparison of adaptive (dashed; from Ref. [1], Figs. 1(a) and 5) and fixed time-step (solid) simulations for the case with a strong mean flow and frozen turbulence. Parameters: $\bar{U}/\sigma_u = 10$, $\eta/L = 10^{-6}$, $r_0/L = 10^{-5}$, 1200 modes, and $\lambda = 0$. The adaptive (fixed) time-step simulation used 5 (100) realizations of the flow with 125 (1) pairs per realization.

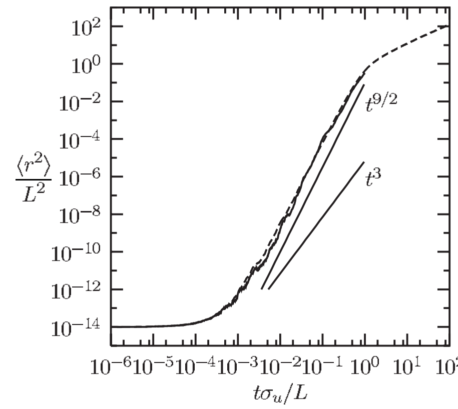

FIG. 2. Comparison of adaptive (dashed; from Ref. [1], Figs. 12 and 13) and fixed time-step (solid) simulations for the case with no mean flow and frozen turbulence. Parameters: $\bar{U} = 0$, $\eta/L = 10^{-8}$, $r_0/L = 10^{-7}$, 1600 modes, and $\lambda = 0$. The adaptive (fixed) time-step simulation used 20 (80) realizations of the flow with 125 (1) pairs per realization.

for $\langle r^2 \rangle$ to reach the point of transition toward a diffusive regime proportional to t. There is significant noise in the new simulations because the computational cost limits the number of pairs we can follow. However, the results are close enough (on the log-log plots) to those with the adaptive time step to confirm that the deductions about power-law behavior made previously were not compromised by the use of an adaptive time step. Note the results with a strong mean flow in Fig. 1 do not show a clear t^6 power law. However, taken with the other evidence given in Ref. [1], which includes simulations with even smaller values of r_0/L and/or η/L, it provides support for t^6 being approached asymptotically as the inertial subrange becomes very long (see discussion in Ref. [1]). Comparisons were also made (not shown) corresponding to the unsteady cases with $\lambda = 5$ in Figs. 10, 15a, and 15b of Ref. [1]. These show a similar degree of agreement between the adaptive and fixed time-step simulations.

Figure 3 shows results with adaptive and fixed small time steps for a case with smaller L/η (10^4) which is similar to the case in Figs. 1, 2, and 4 of Ref. [2]. For the fixed time-step simulation, Ref. [2] uses a slightly smaller time step than Eq. (2). We follow Ref. [2] here, replacing the first term in Eq. (2) by $0.01\varepsilon^{-1/3}k_\eta^{-2/3}$ which, for $L/\eta = 10^4$, can be re-expressed as $0.0724\,\eta/\sigma_u$, where we have taken $C_\varepsilon \equiv \varepsilon L/\sigma_u^3 = 2/3$ as assumed [5] in Ref. [2]. For the adaptive time step simulation, Ref. [2] uses a slightly larger time step than Eq. (1) (it would be the same with $C_\varepsilon = 1$) but we retain Eq. (1) as we are more interested in the performance of our adaptive time step and, in any case, the difference is always less than 15%. In contrast to Ref. [2] we find no significant differences between the simulations. Figure 3(a) shows that $\langle r^2 \rangle$ grows with an exponent somewhere between 3 and 9/2. Figure 3(b) shows the effective eddy diffusivity which has behavior intermediate between $\langle r^2 \rangle^{2/3}$ and $\langle r^2 \rangle^{7/9}$, consistent with Fig. 3(a). We believe these results are a consequence of the limited inertial subrange and we expect the results to

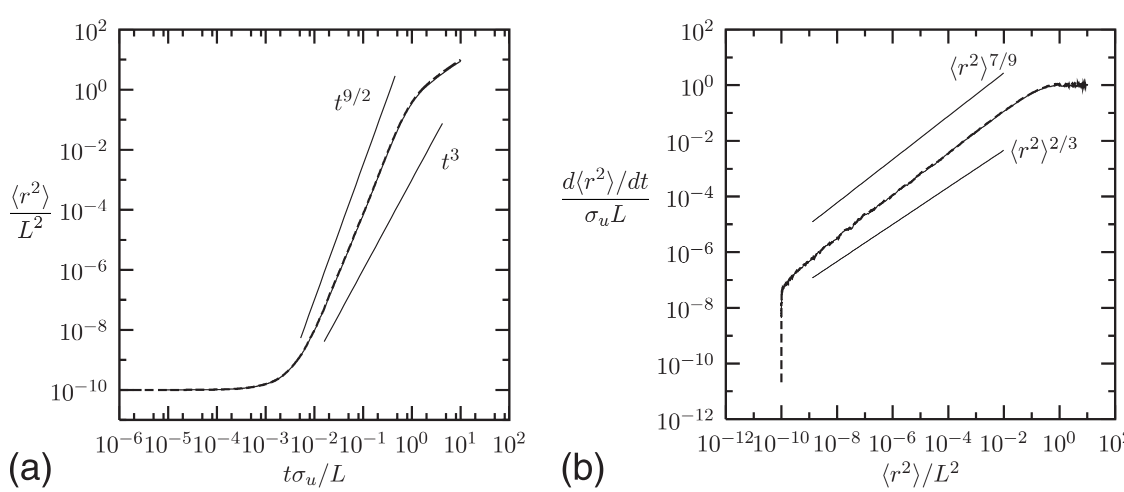

FIG. 3. Comparison of adaptive (dashed) and fixed time-step (solid) simulations for a case similar to that of Ref. [2] (a) shows the evolution of $\langle r^2 \rangle$ while (b) shows the effective eddy diffusivity. Parameters: $\bar{U} = 0$, $\eta/L = 10^{-4}$, $r_0/L = 10^{-5}$, 100 modes, and $\lambda = 0.5$. The simulations used 100 realizations of the flow with 125 pairs per realization.

approach $t^{9/2}$ and $\langle r^2 \rangle^{7/9}$ as $L/\eta \to \infty$ (for fixed λ and r_0/η).

The shape of the $\langle r^2 \rangle$ curve in Fig. 3(a) agrees well with that obtained with a fixed time step in Ref. [2] (the lack of quantitative agreement is probably only due to a scaling error—such an error is likely, at least along the ordinate [5]). In addition the diffusivity results agree quite well with the results in Fig. 2 of Ref. [2] although the latter curve has more curvature at intermediate times. However, we cannot reproduce the significantly different results obtained in Ref. [2] when the fixed time step is replaced by an adaptive time step. In order to eliminate the possibility that our numerical method is not sufficiently accurate, and that this explains the differences from Ref. [2], we repeated the case shown in Fig. 3 using a fourth-order Runge-Kutta method. The results (not shown) showed no significant differences (for both the fixed and adaptive time steps).

To summarize, we have investigated the suggestion made by Ref. [2] that the results on pair dispersion in kinematic simulations which we presented in Ref. [1] may be compromised by the use of an adaptive time step. We have conducted some further simulations with a fixed (small) time step to test this suggestion. These simulations show no significant differences from the adaptive time-step simulations indicating that our results were not compromised in this way.

[1] D. J. Thomson and B. J. Devenish, J. Fluid Mech. **526**, 277 (2005).

[2] D. R. Osborne, J. C. Vassilicos, K. Sung, and J. D. Haigh, Phys. Rev. E **74**, 036309 (2006).

[3] B. Lüthi, J. Berg, S. Ott, and J. Mann, Phys. Fluids **19**, 045110 (2007).

[4] F. Nicolleau, G. Yu, and J. C. Vassilicos, Fluid Dyn. Res. **40**, 68 (2008).

[5] J. C. Vassilicos (private communication).